Frontier Estimation of a Cost Function System Model with Local Least Squares: an Application to Dutch Secondary Education

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Abstract

In this paper we propose a method for efficiency measurement that is based on local estimation in several stages. The method is based on weighted least squares where weights depend on the distance of an observation to all other observations and on the distance to the frontier (efficiency). The new element in the method is that it also includes the information from the cost share equations and includes efficiency in the weighting matrix. The latter is derived from a first stage and implemented in a second stage analysis. An application to a data set of Dutch school boards in secondary education shows that it actually works well. It produces a number of reliable estimates. It also shows a variation in outcomes that would be hard to cover with, for instance, traditional procedures like SFA on a translog cost function.

Key words: local estimation, cost efficiency, scale economies, technical change, education

JEL-codes: C01, D24, I21

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Introduction

Stochastic frontier analysis (SFA) and data envelopment analysis (DEA) are very popular methods to establish the efficiency scores of firms. Both methods have been extensively applied to firms in various industries to get an insight into the relative efficiency of individual firms. The methods have also been applied to compare the performance of departments within firms, and even to compare the performance of countries.

SFA, which was developed by Aigner et al. (1977) and Meeusen & Van den Broeck (1977), is a parametric method. The standard cost or production function is estimated by maximum likelihood methods where the error component consists of random noise and a random efficiency component, which can be separated empirically. Extensive reviews of the SFA approach can be found in Fried et al. (2008), Kumbhakar & Lovell (2000), Coelli et al. (2005), Blank (2000), and Parmeter and Kumbhakar (2014).

DEA is a technique based on linear programming. This technique is derived from early production work by Farrell (1957) and Debreu (1951) and was later formalized using linear programming techniques (Banker et al., 1984; Charnes et al., 1978; Färe et al., 1986). The objective of this approach is to envelop the data points as closely as possible, and to produce the best practice frontier by linking together several line segments. This technique thus identifies the efficient observations and calculates the efficiency scores by measuring the distance to these efficient observations or convex combinations of them.

Both methods have their pros and cons. Each method has been seriously criticized by proponents of the other method for several decades. The critics of SFA focus on the required functional specification of the model and the distributional assumptions about the efficiency component. The critics of DEA focus on the absence of a stochastic component and the difficulty of controlling for environmental variables and deriving economic features like economies of scale and scope and input (or output) substitution.

It is generally recognized that the strong point of SFA is that it takes randomness (measurement and specification errors) into account, whereas the strong point of DEA is the flexibility of the production technology, which does not require some general functional specification. DEA is an observation by observation technique that provides a local estimator.

Only in recent years has there been a tendency in the literature to try to combine the best of both worlds. Kuosmanen (2008) developed a technique that converts a DEA formulation into a stochastic formulation that can be estimated by maximum likelihood techniques. Another approach was developed by Fan et al. (1996), who used standard kernel methods based on maximum likelihood. He applied the stochastic frontier model without the rigidity of a parametric representation of the technology. For an extensive discussion, see Johnson and Kuosmanen (2015) in Ray et al. (2015).

Less criticism is voiced about the fact that SFA has hardly been applied to the full system of equations that can be derived from duality theory. Complicated solutions have been provided by Kumbhakar & Tsionas (2005), based on Bayesian techniques or through the reformulation of the model based on shadow pricing (Blank & Eggink, 2004; Kumbhakar, 1997; Maietta, 2002). Almost all empirical applications of SFA are therefore limited to single equation models.

In this paper we will present a method that is based on the idea of local estimation and includes the information from the cost share equations. A possible answer to the aforementioned issues is applying weighted least squares where weights depend on the distance of an observation to all other observations (more or less lookalikes) and on the distance to the frontier (efficiency). The latter is derived from a first stage and implemented in a second stage analysis. A similar method for deriving efficiency scores in case of a global estimation of a cost function was proposed earlier by Blank & Meesters (2012). To show how the procedure actually works, it is applied to a data set of Dutch school boards in secondary education.

The outline is as follows. Section 2 describes the underlying model and the estimation procedure for estimating the model. In Section 3 the data are described, and in Section 4 the results are presented and discussed. Section 5 concludes the paper.

2. Methodology

The methodology can be applied to a production function, a cost function or any other representation of the production technology and some behavioral assumptions. In this paper we apply a cost function approach.

Since we are only interested in a local estimator of the production technology at a given observation *i* (= *1, .., I*), it suffices to use a first-order Taylor approximation at the given point. However, there is no objection whatsoever to using higher order expansions, except for the number of parameters to be estimated. The cost function therefore can be written as:

$$\ln(C) = a_0 + \sum_m^M b_m \ln(y_m) + \sum_n^N c_n \ln(w_n) + \sum_k^K d_k \ln(z_k) + h_1 time \quad (1)$$

With:

y_m = output *m*;

w_n = input price *n*;

z_k = environmental characteristic *k*;

time = trend;

b_m, c_n, d_k, h_1 parameters to be estimated;

In addition, we also estimate simultaneously the cost share equations as:

$$sh_n = c_n \quad (n = 1, .., N) \quad (2)$$

With:

sh_n = cost share of input *n*;

The system of equations will be estimated with weighted nonlinear least squares. The weights are based on the distance of the reference observation to the other observations and to the frontier. The idea behind this approach is, to put it simply, that we would like to base the estimates on efficient neighbors as much as possible. The extent to which this is possible is an empirical matter. The weight function, for instance, can be described as:

$$weight = eff * \text{norm}\left[\frac{dist}{k \cdot \sigma_{tot}}\right] \tag{3}$$

With:

eff = efficiency;
$dist$ = distance;

σ_{tot} = standard deviation of distance measure;
k = scaling parameter;
norm(.) is the normal density function.

In order to obtain a distance measure that does not depend on the unit of measurement, all variables are standardized on their means. Then the distance is measured by the Euclidean distance:

$$dist = \sqrt{\sum_{\mathrm{m}}^{\mathrm{M}} (y_m - y_m^*)^2} \tag{4}$$

With:
dist = average distance to the reference observation;

y_m^* = value of output m of the reference observation.
$weight$ = weight attached to an observation;
k is a (fixed) parameter comparable to bandwidth
σ_{tot}^2 = sum of variances of y_m

Then the following set of equations of a cost function model is estimated:

$$\epsilon_0 = \text{weight} * \left[\ln\left(\mathrm{C}^{\text{obs}}\right) - \ln(\mathrm{C})\right] \tag{5}$$

$$\epsilon_{\mathrm{n}} = \text{weight} * \left[\text{sh}_{\mathrm{n}}^{\text{obs}} - \text{sh}_{\mathrm{n}}\right] \tag{6}$$

For each observation i =*1*, .., I we apply (local) least squares (LLS). After each least squares estimation, we set $eff_i = \exp(-\epsilon_i)$. The efficiency scores will be used in the next stage to set weights (along with the distance to the reference point).

Summarizing:

The procedure is conducted in several stages $s = 1, \ldots, S$ and stops at iteration S when the efficiency scores change less than some threshold value (= 0.01). At $s = 1$, the vector of weight parameters is set to 1.

At each stage, weighted least squares is applied for each DMU separately to a cost function model, consisting of a cost function and cost share equations. The weights are based on the distance (*dist*) between a DMU and the DMU under investigation and the cost efficiency (*eff*) of the DMU: The larger the distance, the smaller the weight, and the larger the efficiency, the larger the weight.

Each separate LLS for a DMU provides an estimate of the efficiency parameter, which can be used in the next stage of the procedure to set the weight parameter. Note that the efficiency parameter varies only per stage and the *dist* parameter per LLS analysis.

At $s = S$, economic outcomes can be presented, such as scale elasticity, marginal cost, technical change, and cost efficiency scores.

3. Data

Production

The different types of schools in secondary education require different educational processes and consequently lead to different costs. For example, a teacher who teaches students in the final year of pre-academic education is generally more expensive than a teacher for students in the first year of vocational training. Therefore, the production cannot be captured in one number. Production indicators are based on the different types of education and grades. We therefore distinguish:

- Grade 1 and 2 of all types of education;
- Grade 3-4 VMBO (vocational training);
- Grade 3-6 HAVO and VWO (general higher and pre-academic).

Quality in education is generally difficult to measure. In order to take the quality of education into account, passes to next grades and examination results are included. The influence of the initial skills of pupils on quality measures are taken into account by including the so-called school recommendation at the beginning of a pupil's school career.

The resources

The resources used can be divided into five categories or types of costs:

- Teaching personnel;
- Administrative personnel;
- Executive board and management;
- Housing (excluding rent);
- Material supplies.

We exclude capital cost because for most institutions, the local government is responsible for providing the school buildings. For a meaningful comparison with institutions owning their own school buildings, rent and amortization of buildings are therefore excluded.

Resource prices

The relative prices of the staff categories differ by region and year. Averaging personnel costs per full-time equivalent over regions and years by a regression analysis provides a labor price for each staff category for each region in a certain year.

The prices for housing and material are assumed to be equal for all educational institutions and thus only vary over the years. Since housing costs are merely building-related costs such as energy and cleaning, the energy price indices of Statistics Netherlands are used for the housing costs. For the material costs, the consumer price index of Statistics Netherlands is used.

Data resources, data checks and manipulations

For the analyses, we used different databases. The number of pupils was taken from the public files of the Office of Education (DUO) and the Ministry of Education, Culture and Science. The numbers on education returns were supplied by the Education Inspectorate. The staff numbers and salary data were also provided by DUO. Finally, the price development of energy and consumer goods and services was collected by Statistics Netherlands. The period for which all necessary data are available is 2007-2010.

Data checks and manipulations

We applied a number of checks and manipulations to these data (for details, see Urlings & Blank, 2012). A statistical description of the data is given in Table 3 for the year 2010.

Table 1 Statistical description variables in analysis, 2010.

Variable	*Mean*	*Std. Error*	*Minimum*	*Maximum*
Grades 1-2[a]	1148.5	821.5	185.1	5783.3
Vocational training grades 3-4[a]	515.8	375.7	76.5	2501.3
General education grades 4-6[a]	650.7	491.4	93.1	3095.3
Total cost (x € 1000)	19179	14358	5100	105563
Cost share board/management	0.05	0.02	0.00	0.16
Cost share administrative personnel	0.09	0.04	0.00	0.24
Cost share teaching personnel	0.65	0.05	0.44	0.81
Cost share housing	0.07	0.03	0.02	0.28
Cost share material supplies	0.14	0.03	0.06	0.29
Price management (€)	100014.3	4960.3	88393.0	110339.0
Price administrative personnel (€)	46388.9	3872.6	37756.0	52967.0
Price board/management (€)	65278.5	3937.8	57210.0	74661.0
Price housing (€)	358.5	14.4	342.0	380.5
Price material supplies (2007=100)	104.6	1.8	101.6	106.7

[a] Corrected with pass rate

Secondary education statistics

In 2010, the average secondary school in the Netherlands had 3,300 pupils. Of these, 38% were in the first two grades, 19% in junior vocational education (vmbo), 35% in senior general secondary education (havo) or pre-university education (vwo), and 8% in other education (practical education, primary education or senior vocational education).The costs can be divided across five categories:

- teaching staff (65%);
- administrative staff (9%);
- management (5%);
- accommodation (6%);
- material supplies (15%).

There is a strong variation in the scale of the educational institutions. Half of the educational institutions have fewer than 2,100 pupils and costs of under 17.5 million euros. The largest educational institution has over 62,000 pupils and costs totalling 482 million euros.

4. Results

The outcomes are presented by graphs. Figures 1 to 3 present the marginal costs of different types of pupils who passed. The marginal cost gives a first indication of the plausibility of the estimates. The marginal cost of an undergraduate pupil has more or less a normal distribution around € 8,000 with a limited variance. Marginal costs of pupils in vocational training are higher, distributed around € 10,000. This distribution is skewed to the right. A number of schools tend to have higher marginal costs than the modus. The marginal cost of pupils in general education (figure 3) has a distribution around € 6,000. This distribution is skewed to the left, indicating that there are relatively more schools with lower marginal costs than schools with high marginal costs. The outcomes make sense, since it is known that vocational training is more expensive than general education due to higher material costs (machinery, etc.). General education is less expensive than undergraduate education due to the substantially lower number of teaching hours in the graduate phase of education.

Figure 1 Estimated marginal costs of undergraduate pupils (corrected for passes)

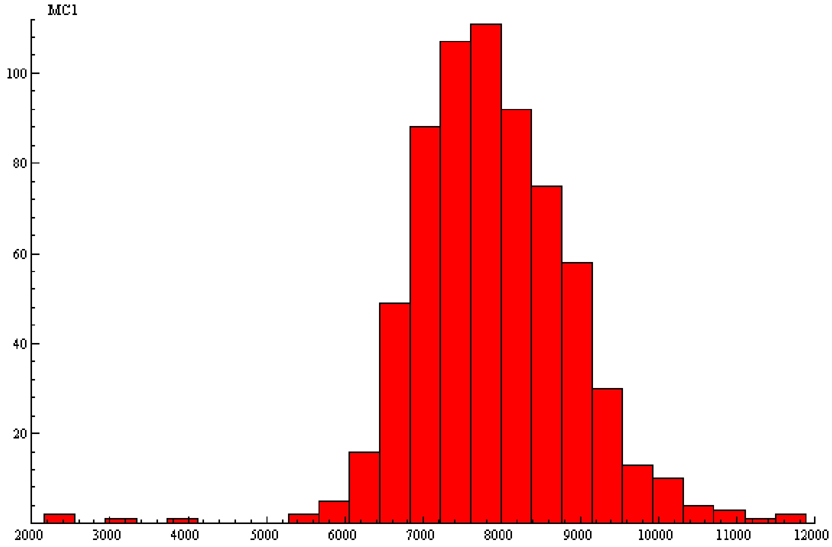


Figure 2 Estimated marginal costs of pupils in vocational training (corrected for passes)

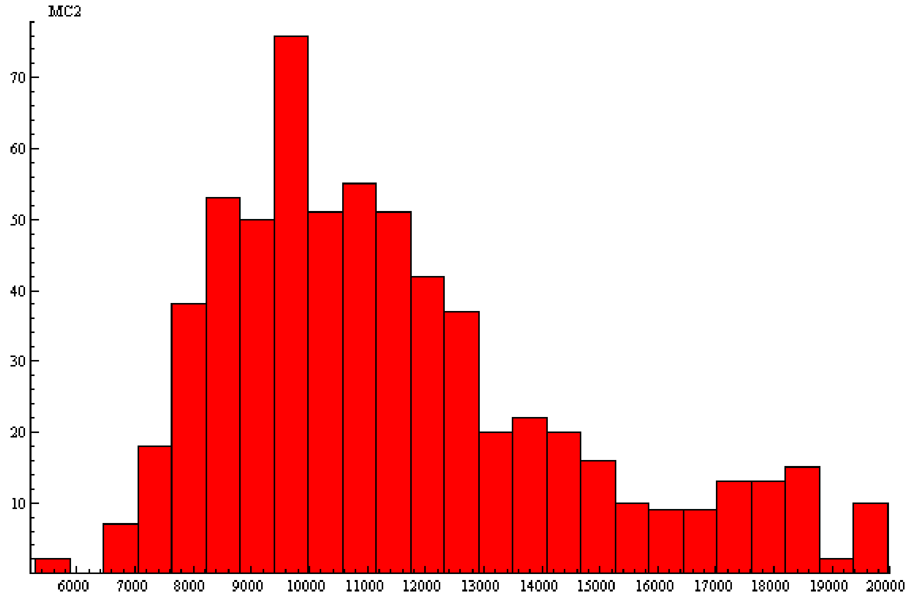


Figure 3 Estimated marginal costs of pupils in general education (corrected for passes)

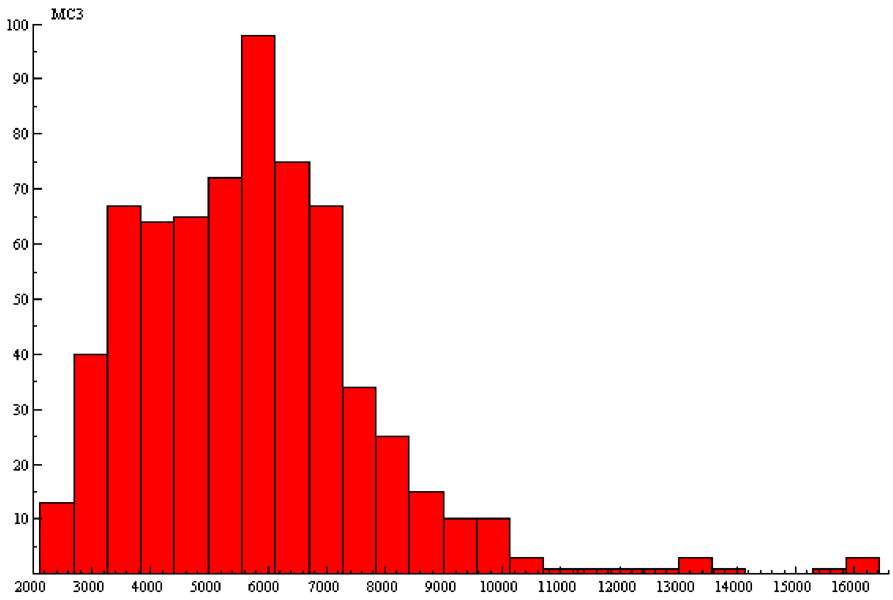


Figure 4 represents the estimated cost flexibility of each school board (the red + signs). Each school board is reflected by its size, expressed in terms of a number times the average size. So two, for instance, reflects a school board that is twice the size of the average school board. The green and purple lines represent the lower and upper bounds of the 95% confidence interval. Outcomes less than one indicate economies of scale, and outcomes greater than one indicate diseconomies of scale. From figure 4 we can conclude that school boards less than two times the size of the average school board face economies of scale, whereas school boards greater than three times the average size face diseconomies of scale. The optimum size (neither economies nor diseconomies of scale) lies between two and three times the average size.

Figure 4 Economies of scale (cost flexibility with 95% CI)

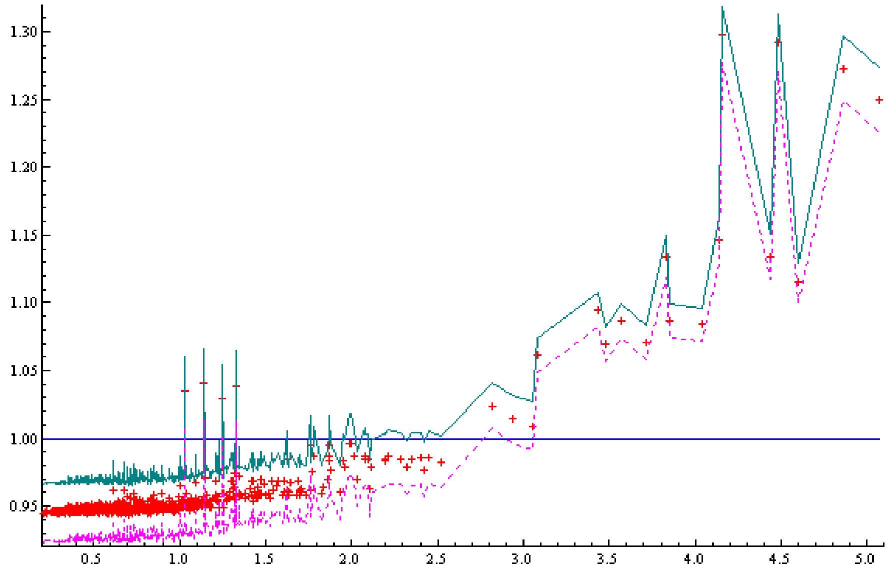


Figure 5 represents the distribution of the efficiency scores. It shows that the majority of the schoolboards are efficient or close to efficient. This is due to the fact that only school boards are regarded as suitable references when they have a rather similar production profile. Observations with deviated production profiles receive a low weight in the estimation procedure. Sensitivity analysis based on local estimation with a wider bandwidth may result in a different efficiency pattern. However, it shows that estimation with different bandwidths leads to almost identical distributions of efficiency scores.

Figure 5 Efficiency scores

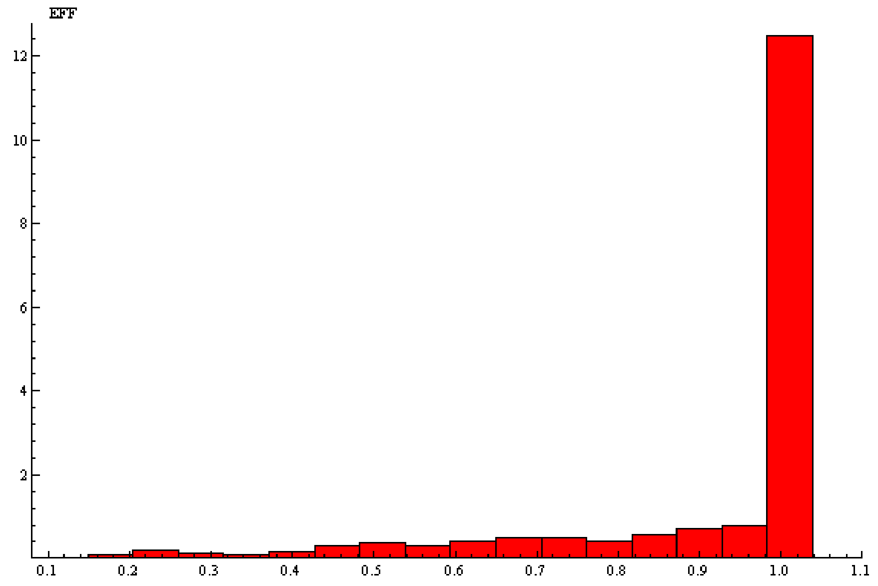
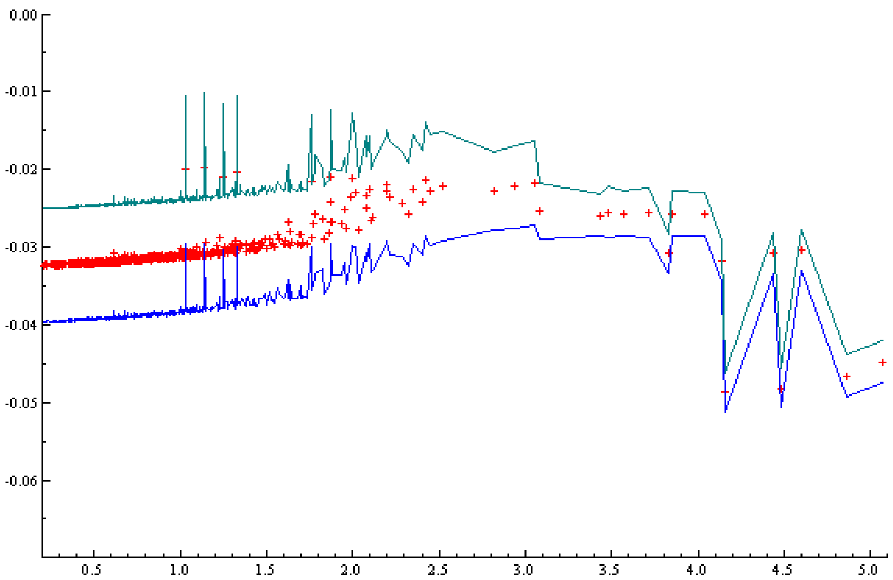


Figure 6 displays technical change. The mean is represented by the red dots, while the 95% confidence intervals are represented by the upper (green) and lower bounds (blue). It shows that technical change is significantly negative for all schools, implying a loss of productivity of about 3% annually. It also shows that for a few larger school boards, technical change is even more negative.

Figure 6 Technical change (with 95% CI)



Further, we checked the outcomes on the conditional mean of the cost shares. They show a very consistent pattern with respect to size.

We also conducted some sensitivity analyses by varying the bandwidth (k parameter in equation 3). We calculated the outcomes for different k (= 0.25, 0.50 and 1.00). Table 2 summarizes the outcomes by presenting a test of the mean difference in the efficiency scores, the estimates of technical change and the estimated cost flexibilities.

Table 2 Test results of mean differences for varying bandwidth (*k* = 0.25, 0.5 and 1)

Test of mean difference	*Mean*	*Std. Error*	*T-test*
Efficiency k=1vs k=0.5	-0.001	0.000	1.58
Efficiency k=0.5 vs k=0.25	-0.002	0.001	-3.34
Technical change k=1 vs k=0.5	-0.000	0.000	-1.91
Technical change k=0.5 vs k=0.25	-0.001	0.000	-16.30
Cost flexibility k=1vs k=0.5	0.002	0.003	0.67
Cost flexibility k=0.5 vs k=0.25	-0.008	0.001	-8.05

From Table 2 we conclude that the differences between the different (point) estimators are relatively low. In case of the efficiency scores, a larger bandwidth (corresponding to a larger k) corresponds to lower efficiency scores. The differences, however, are very small and in one case not significantly different from zero. Note here that efficiency scores are about 95% on average. For technical change, the differences are also very small. In spite of the fact that the mean differences are significant, the differences are still modest. The same holds for the cost flexibility. The differences are less than 1%. Larger bandwidths correspond to a slight increase in cost flexibility, implying that economies of scale cease to exist at lower production levels. Further, it is striking that, although the differences are very small, all differences are significant at the 5%-level with respect to the test k = 0.5 versus k=0.25.

5. Conclusion

In this paper, we presented a method for efficiency measurement that is based on the idea of local estimation in several stages. The new element in the method is that it also includes the information from the cost share equations and includes efficiency in the distance measure. The method is based on weighted least squares, where weights depend on the distance of an observation to all other observations (more or less lookalikes) and on the distance to the frontier (efficiency). The latter is derived from a former stage and implemented in a next stage analysis. An application to a data set of Dutch school boards in secondary education shows that it works well. The approach produces a number of reliable estimates. It also shows a variation in outcomes that would be hard to cover with, for instance, traditional procedures like SFA on a translog cost function. It therefore seems that the proposed approach could be an interesting alternative to standard frontier techniques. This approach adds more flexibility to the modeling of the production technology.

Nevertheless, a number of issues still need to be addressed. The set of weights is based on a distance measure and the efficiency score. For the distance measure, a traditional Euclidean measure is used, whereas efficiency scores are assumed to be directly related to the estimated errors in the first stage of the procedure. Sensitivity analysis certainly shows that varying the bandwidths has only limited effects on the outcomes, but we have presented here only one application. Therefore, more research on the effect of alternative distance and efficiency measures is required.

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